# 7. DISTRIBUTION NETWORK OPTIMIZATION USING GRAVITY POINT

The chapter discusses the problem of optimizing the logistics network using Gravity Point (Gravity Model). The supply chain network transforms raw materials into final products and, of course, delivers them to end customers (consumers). It includes different types of objects. Supply chain network design (SCND) is an important issue related to supply chain management (SCM). The most important issues discussed in this chapter include:

- Supply chain network,
- Distribution network,
- Gravity Point.

## 7.1. Introduction

**Supply chain network design** (SCND) is an important issue related to supply chain management (SCM). A supply chain is understood as a complex network of enterprises and facilities, most of which are distributed over a large geographical area. This supply chain should synchronize a number of interrelated activities through a network.

The supply chain network transforms raw materials into final products and, of course, delivers them to end customers (consumers). It includes different types of objects. Planning and designing a supply chain network therefore focuses on identifying the number and types of individual links and coordinating activities between them. Typical links in a supply chain network consist of suppliers and subcontractors, manufacturing and assembly plants, distribution centers, warehouses, and customers (Govindan et al., 2017). Typical material flows take place from suppliers to customers. Reverse flows can also be distinguished (so-called reverse logistics). You should also remember about the need to plan and design flows and deal with problems related to many variants/types of products. Analysis related to the location of a given facility in the supply chain is an important issue for the operation of this chain and costs.

## 7.2. Logistics network

The complexity of the supply chain network is important and influences planning decisions along with classic location allocation decisions to achieve an integrated system (Govindan et al., 2017).

Considering the three levels of decision-making, at the **strategic level**, supply chain decisions need to be made such as: (1) number, (2) location, and (3) capacity of facilities. Strategic decisions typically have a time horizon of about three to five years. Strategic decisions regarding the design of the logistics network influence the effectiveness of customer demand service. Design decisions cannot be made without considering the impact on operational decisions. **Tactical decisions** typically span from three months to three years. For example, pricing decisions are typically placed at the tactical planning level. **Operational decisions** (e.g., vehicle routing decisions) often range from one hour to one trimester (Govindan et al., 2017). Of course, the scope of decisions made may depend on the nature of supply chains.

The selection of the best location for a business facility may be considered in terms of general or specific location. The general location defines a certain area where a given economic facility is to be located.

There are many factors that influence the location of an object in the supply chain. These are among others:

- sources of raw materials and location of markets for production materials (mainly raw materials, components),
- industrial traditions of the region, including accessibility to suppliers and customers (particularly important for the activity of intermediate links),

- labor force (employment opportunities, remuneration, availability, level of qualifications),
- possibilities of supplying energy factors,
- tax regulations and administrative restrictions,
- climate and terrain conditions,
- availability of roads and transport points,
- characteristics of population, socio-political relations,
- infrastructure characteristics (roads, schools, communication),
- possibility of expanding the facility.

However, a detailed location indicates a specific property or area where the facility is to be built. The choice of a detailed location is related to, for example, its technical infrastructure, availability of transport infrastructure (local roads), as well as the local development plan.

In more detail, you should also take into account:

- wage rates in neighboring plants,
- communication options for the crew and travel fees,
- possibility of purchasing the desired plot in a selected region,
- roads, highways and land development with water and gas networks,
- safety zones for odors, noise and pollution,
- terrain enabling the construction of production and auxiliary facilities, parking lots
- possibility of future expansion in accordance with the needs of the production process and the requirements of architectural and construction authorities.

The scope of detailed localization is not covered in this study.

It is important to remember that supply chains operate in a changing environment. It often happens that facilities are closed, opened or reopened more than once within the established planning horizon. Market dynamics force another decision to be made, i.e. the issue of increasing, decreasing or transferring the production capacity of facilities in the logistics network. Another important issue is any type of **disruption** to the functioning of supply chains. A supply chain disruption is an event that may occur in part of the supply chain due to, for example, natural disasters (e.g. earthquakes and floods) and intentional or unintentional human actions (e.g. wars and terrorist attacks). It is identified as an event that interrupts the flow of materials in supply chains, causing the flow of goods to suddenly stop. Even a small disruption can have a devastating impact on the functioning of supply chains as it cascades through the chain (Grzybowska & Stachowiak, 2022). And since supply chains are complex and heterogeneous structures, they are vulnerable to threats and difficult to manage.

The distribution network, often the downstream part of the SC network, consists of product streams from warehouses to customers or retailers. Designing such a network requires solving two difficult combinatorial optimization problems, including determining the location of the facility and the routes of vehicles to serve customers.

## 7.3. The concept of using the gravity model in a logistics network

Building a reasonable logistics network is the key to the development of regional logistics. The gravity model is derived from Newton's gravity - let us recall: the law of gravity is the law of universal gravitation, the purpose of which is to describe the force with which bodies attract each other.

Gradually, the concept of applying the gravity model was applied to other studies, areas and fields by analogy with physics. In its later expansion, in addition to proving the existence of the gravity model theory itself, it was applied to many disciplines. Among them, the most widely developed research is related to trade, urban spatial connections and logistics:

- Reilly was the first to use the gravity model to study relationships between cities (1929),
- Stewart proposed the concept of the gravity model (1948),
- Tinbergen introduced the gravity model (GM) to international trade (1962),
- Huff proposed the use of the gravity model to estimate market share (1963),
- Bergstrand clarified the supply side of economies, indicating the theoretical foundations of the relationship between the endowment of production factors and trade with Constant Elasticity of Transformation (CET) (1989),

- Kong et al examined the design of green space networks using a gravity model (2010),
- Duanmu et al developed a coupled gravity model and genetic algorithm to study charge distribution (2012),
- Puertas et al. used the gravity model to analyze the logistics network - estimating the logistics efficiency index (2014),
- Zhu & Fan used the gravity model to study the intensity of logistics connections in inland regional logistics (2017).

**Distance** in Newton's model is an approximation of resistance to motion, i.e. a factor that weakens the force of attraction. This means that the more distant partners are from each other, the less intense their mutual trade is. The main reason for this is the existence of trade transaction costs, which increase with increasing geographical distance. These costs include, among others: transport costs or cargo insurance (Bułkowska, 2018).

Geographical location has always been a factor determining business activity. The meaning and possibilities of transport have changed. Geography is one of the main sources of trade costs, that is, the spatial characteristics of countries that influence their domestic and international transportation costs. Features that are taken into account include geographic distance between facilities or countries. In the case of country analysis, the analysis includes answers to the questions: do the countries have a common border?, are they landlocked countries?, are they island countries? Intuition suggests that greater geographic distance, lack of a common border, and/or greater distance from a trading partner negatively impact transportation costs. Therefore, it has a negative impact on international trade. These consequences can be mitigated through infrastructure development such as the creation of highways, tunnels, airports and ports (Azmi, et al., 2024).

One of the factors in the location of business facilities is the proximity of the sales market. This neighborhood takes on a new and crucial meaning. It is becoming an asset again after the experience of the COVID-19 pandemic and in relation to improving the resilience of supply chains to disruptions.

This applies especially to companies that:

- produce or supply perishable goods,
- are characterized by high price elasticity of supply or services offered,
- produce products that are characterized by high demand variability,
- produce or transport goods that are burdensome to transport.

## 7.4. Typical decision-making process regarding the location of a facility in the supply chain

In the short term, the manager must operate within the constraints imposed by the location. However, in the long term, location becomes a variable and the manager may make decisions to change the location in order to meet the requirements of customers, suppliers or changes imposed by competitors.

External factors influencing the motivation to analyze the location of a new facility or change the location of a facility are:

- expansion into new markets,
- shifting of residential clusters,
- threats from competition,
- emergence of new supply markets.

The location should meet two criteria: quantitative (cost) and qualitative. Quantitative criteria are considered first. The object location pattern has the form:

$$\boldsymbol{C} = \frac{\sum \boldsymbol{r_i} \cdot \boldsymbol{d_i} \cdot \boldsymbol{S_i} + \sum \boldsymbol{R_i} \cdot \boldsymbol{D_i} \cdot \boldsymbol{M_i}}{\sum \boldsymbol{r_i} \cdot \boldsymbol{S_i} + \sum \boldsymbol{R_i} \cdot \boldsymbol{M_i}}$$

where:

$C$ – center of mass

$d_i$ – distance from point 0 on the grid to the location of the source of raw material $i$

$D_i$ – distance from point 0 on the grid to the point of location of the source of sales market $i$

$S_i$ – weight volume of raw materials purchased from supply sources $i$

$M_i$ – weight volume of finished products sold on market $i$

$r_i$ – transport rate for finished product $i$

$R_i$ – transport rate for raw material $i$.

This is explained by the formula used in Excel:

**center of mass = a + b / c+d**

**a = SUM [transport rate for raw material$_{(i)}$ * distance from point 0 on the grid to the point of location of the source of raw material$_{(i)}$ * weight volume of raw material$_{(i)}$]**

**b = SUM [transport rate for the finished product$_{(i)}$ * distance from point 0 on the grid to the point of location of the source of the market$_{(i)}$ * weight volume of the finished product$_{(i)}$]**

**c = SUM [transport rate for raw material$_{(i)}$ * weight volume of raw material$_{(i)}$]**

**d = SUM [transport rate for finished product$_{(i)}$ * distance from point 0 on the grid to the point of location of the source of the market$_{(i)}$]**

## 7.5. Disaggregated and aggregated gravity models

There are many variants of the gravity model that can be used to simulate flows between retailers and consumers. The choice of model depends on the purpose of its use and on the data available to fit the model. When choosing a gravity model, the level of aggregation is also an important factor. Shopping interactions between consumers and retailers can be represented in a **disaggregated model** that estimates consumer behavior. Can also be presented in an **aggregated model**. In this variant, retail outlets in the zone are assessed collectively (Schlaich, 2020). In aggregate models, the characteristics of individual stores and the exact distances between the consumer and the retailer disappear. On the other hand, aggregation into zones significantly reduces the complexity of the model as the set of destinations decreases.

Of all models of spatial interaction in retailing, Huff's (1963) gravity model is one of the most widely used. In its initial form, this model calculates patronage probabilities depending on store size and transportation distance.

In gravity models, an important issue is to determine the variable describing the "power of mutual attraction" of trading partners, i.e. the model explained (dependent) variable. Gravity models provide geographers and economists with a flexible analysis tool.

## 7.6. Balanced gravity model

The balanced center of gravity method is used to determine the location of a single economic facility (e.g. warehouse). It takes into account sources of demand of various importance and location. The location is determined using coordinates (X, Y), which indicate the position of the point on the map. The importance is related to, for example, the volume of deliveries, the number of people living in a given location or the sales value. You can also use another indicator, it is important that it is properly adjusted to the situation. The described method uses weighted supply point coefficients, thus generating a point on the map marked with coordinates.

For the weighted centroid method, use the model:

$$\boldsymbol{X^*} = \frac{\sum \boldsymbol{W_i \cdot X_i}}{\sum \boldsymbol{W_i}}$$

$$\boldsymbol{Y^*} = \frac{\sum \boldsymbol{W_i \cdot Y_i}}{\sum \boldsymbol{W_i}}$$

where,

$X_i, Y_i$ – coordinates of the i-th source of demand

$W_i$ – weight of the i-th source of demand

The weighted coordinates $(X^*, Y^*)$ calculated using the model indicate the appropriate location of the supply point, taking into account the importance (importance) of individual demand sources.

This is explained by the formula used in Excel:

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**coordinates of the supply point (X) = SUM [(weighted indicator of the demand source$_{(i)}$ * coordinates X$_{(i)}$)] / SUM coordinates X$_{(i)}$**

**supply point coordinates (Y) = SUM [(weighted indicator of demand source$_{(i)}$ * Y coordinates$_{(i)}$)] / SUM Y coordinates$_{(i)}$**

The balanced center of gravity method allows you to determine the location of one economic facility in a selected geographical area. The method is simple to use and comes down to determining two parameters on a geographical grid.

An extension of this method is the model:

$$\boldsymbol{Coordinates}_{\,(X,Y)} = \frac{\sum \boldsymbol{r_i} \cdot \boldsymbol{d_i} \cdot \boldsymbol{S_i} + \sum \boldsymbol{R_i} \cdot \boldsymbol{D_i} \cdot \boldsymbol{M_i}}{\sum \boldsymbol{r_i} \cdot \boldsymbol{S_i} + \sum \boldsymbol{R_i} \cdot \boldsymbol{M_i}}$$

where,

$Coordinates_{\,(X,Y)}$ – center of gravity

$r_i$ – transport rate for finished product *i*

$d_i$ – distance from point *0* on the grid to the location of the source of raw material *i*

$S_i$ – weight volume of raw materials purchased from supply sources *i*

$R_i$ – transport rate for raw material *i*

$D_i$ – distance from point *0* on the grid to the point of location of the source of sales market *i*

$M_i$ – weight volume of finished products sold on market *i*

Calculations are performed for vertical and horizontal coordinates.

This is explained by the formula used in Excel:

**Counter(X) = SUM (transport rate for the finished product$_{(i)}$ * distance from point 0 on the grid to the location point of the raw material source$_{(Xi)}$ * weight volume of raw materials purchased from the supply sources$_{(i)}$) + SUM (transport rate for the raw material$_{(i)}$ * distance from point 0 on the grid to the point of location of the source of the market$_{(Xi)}$ * weight volume of finished products sold on the market$_{(i)}$)**

**Denominator = SUM (freight rate for finished product$_{(i)}$ * weight volume of raw materials purchased from supply sources$_{(i)}$) + SUM (freight rate for raw material$_{(i)}$ * weight volume of finished products sold on the market$_{(i)}$)**

**supply point coordinates (X) = Numerator$_{(X)}$ / Denominator**

**Counter$_{(Y)}$ = SUM (transport rate for the finished product$_{(i)}$ * distance from point 0 on the grid to the location point of the raw material source$_{(Yi)}$ * weight volume of raw materials purchased from the supply sources$_{(i)}$) + SUM (transport rate for the raw material$_{(i)}$ * distance from point 0 on the grid to the point of location of the source of the market$_{(Yi)}$ * weight volume of finished products sold on the market$_{(i)}$)**

**Denominator = SUM (freight rate for finished product$_{(i)}$ * weight volume of raw materials purchased from supply sources$_{(i)}$) + SUM (freight rate for raw material$_{(i)}$ * weight volume of finished products sold on the market$_{(i)}$)**

**supply point coordinates (Y) = Numerator$_{(Y)}$ / Denominator**

## 7.7. Gravity model in international trade

Tinbergen (1962) was the first to provide an intuitive explanation of bilateral trade flows in international trade. His discoveries laid the foundation for the modern gravity model, which assumes that trade between nations is directly proportional to the size of their economies and inversely proportional to the costs of trade. This should be understood as follows:

- larger countries are expected to trade more,
- countries that are further apart are expected to trade less (possibly due to higher trade costs).

Since then, the model has been widely used in the industry literature to explain international trade flows. Due to the effectiveness of the gravity model in trade research, we have seen a significant increase in the use of the gravity model to assess various aspects of international trade (Azmi, et al. 2024).

$$\boldsymbol{X_{ij} = \alpha_i + \beta_1 \cdot GDP_i + \beta_2 \cdot GDP_j + \beta_3 \cdot TC_{ij} + \mu_i}$$

where:

$X_{IJ}$ – flow in international trade from country $I$ to country $J$

$GDP_i$ i $GDP_j$ – gross domestic product of the country of origin and country of destination

$TC_{IJ}$ – the cost of trade between two countries, estimated by the geographic distance between the capital cities

$\mu_i$ – random error

$\alpha_i$ – model intersection point

$\beta_1, \beta_2, \beta_3$- coefficients measuring the impact of explanatory variables.

This is explained by the formula used in Excel:

**trade flow = intercept + coefficient$_{(1)}$ * income of exporting country + coefficient$_{(2)}$ * income of importing country + coefficient$_{(3)}$ * cost of trade between two countries + random error**

Various variants of the presented model are also known. Below is one of them:

$$\boldsymbol{X_{ij} = \beta_0 + \beta_1 \cdot y_i + \beta_2 \cdot y_j + \beta_3 \cdot n_j + \beta_4 \cdot n_i + \beta_5 \cdot d_{i,j} + \beta_6 \cdot D_{ij} + \mu_{ij}}$$

where:

$X_{ij}$ – rade flow (export or import from country *i* to country *j)*

$y_i$ – income of exporting country *i*

$y_j$ – income of the importing country *j*

$n_j$ – population of country *i , j*

$d_{i,j}$ – distance between countries *i* i *j*

$D_{ij}$– a dummy variable with the value 1 if countries i and j are members of specific preferential trade areas, and 0 otherwise

$\beta_0$– represents the intersection point

$\beta_1 - \beta_6$ – coefficients$y_i, y_j, n_j, n_i, d_{i,j}, D_{ij}$ respectively

$\mu_i$ – random error.

This is explained by the formula used in Excel:

**trade flow = intercept + coefficient(1) * income of exporting country + coefficient(2) * income of importing country + coefficient(3) * population of country j + coefficient(4) * population of country i + coefficient(5) * distance between countries + coefficient(6) * dummy + random error**

## 7.8. Gravity model of locating competitive objects

Most competitive facility location models assume that all available purchasing power is shared among competing facilities.

The leitmotif of all competitive location models is the existence of interconnections between four variables: purchasing power (demand), distance, attractiveness of the facility and market share. The first indicated variables are independent variables, while market share is a dependent variable.

Each competing facility, e.g. a commercial facility, has a "sphere of influence". It is determined by his level of attractiveness. More attractive objects have a larger radius of their sphere of influence. The purchasing power expended by the consumer in the sphere of influence of several objects is divided equally between competing objects (Drezner & Drezner, 2016).

Competitive location models have a number of applications, e.g. they enable the location of shopping centers, stores (e.g. grocery stores, specialty stores - household appliances; footwear; bookstores; computers, jewelry...), restaurants (fast food, cafes, ice cream parlors...), gas stations, bank branches and other.

## 7.9. Gravity model for intercontinental supply chain

Gravity models can serve as suitable assessment tools for estimating cargo delivery to ports, with time and distance costs playing an important role (Wang & Li, 2021). To analyze the interaction patterns of retail regions associated with different agglomerations, Reilly created a gravity model of goods flows as:

$$\boldsymbol{X_{ij}} = \boldsymbol{\alpha} \frac{\boldsymbol{P_i \cdot P_j}}{\boldsymbol{d_{ij}^2}}$$

where:

$X_{ij}$ – flow in the supply chain

$d_{ij}$ – spatial distance

$P_i, P_j$ – population in the place of origin $i$ and destination $j$

$\alpha$ – gravity coefficient, constant equal to 1.

This is explained by the formula used in Excel:

**flow in the supply chain = gravity coefficient * population in the place$_{(i)}$ * population in the place$_{(j)}$ / spatial distance$^2$**

In this model, the location of all nodal cities is known. The gravitational pull between cities can be determined by the city's size and spatial distance.

# Chapter Questions

1. What external factors influence the decision to relocate a facility?
2. What are the main advantages and limitations of using different variants of the gravity model to simulate flows between retailers and consumers?

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