## **10.TRANSPORT OPTIMIZATION**



This chapter is devoted to the most important issues related to transport optimization. It includes:

- basic definitions of the transport system,
- the nature and importance of transport system optimization,
- description of the application of the transport issue in practice.

#### 10.1. Introduction

Transport and forwarding processes play a dominant role in the distribution phase. It is important to note that their increasing importance generates the need for innovations concerning the organization of the movement of cargo using appropriately selected means of transport and mode of transport (Krawczyk, 2001). Methods and tools are therefore being sought to provide precise answers to key questions regarding transport and forwarding processes. One may wonder whether any changes need to be made and, if so, what financial results will result? The challenge, as well as the need of the modern company, is to combine the economic benefits of maintaining a high quality of customer service with the reduction of transport costs, which is also a function of customer service. In doing so, the issue of cost minimization takes on a strategic dimension (Christopher, 2000).

Transport belongs to the field of production of material services, performs transportation of people and goods, provides distribution and supply of raw materials and products of industry and agriculture to all regions of the world. at home and abroad. The main task of transport is to fully and timely meet the transport needs of the national economy and population, to increase the efficiency and quality of the transport network. Given the leading role of transport in the market economy, transport management is assigned to a separate field, called transport logistics. Transport logistics includes a number of elements, the main ones being (Yahiaoui, 2019; Vakulenko & Evreenova, 2019):

loads,



- consolidation stations,
- transport hubs,
- transport network,
- rolling stock,
- handling equipment,
- logistics process participants,
- transport containers,
- packaging.

The main reserves for improving the transport and logistics process lie in the rational organization of supply chain participants' interactions, coordination of their interests and the search for mutually beneficial and appropriate solutions. Advances in information technology can significantly improve the efficiency of transportation logistics, and information and IT support have a rightful place among key logistics functions (Liu, Zhang & Wang, 2018; Sun, et al., 2019).

Advances in information technology have contributed to increased transport efficiency. The use of the latest information technology makes it possible to automate all the IT activities of transport companies that are involved in the processes of organising freight traffic. The automation of transport logistics provides increased efficiency and optimization of transport. With the introduction of automated routing, billing and planning systems in transport enterprises, transport logistics reaches a new level (Dekhtyaruk, et al. 2021).

# 10.2. The nature and importance of transport system optimization

Transport system optimization research and work is linked to important transport policy issues, playing an important role in the development of transport economics theory. The development of optimization work and research is stimulated by the economic practice of transport, indicating the most relevant issues to be solved, as well as determining the scope and directions of methodological research. On the other hand, the topic of transport system optimization has inspired researchers to implement the achievements of systems theory and cybernetics to solve economic transport problems. This has contributed to favourable



developments in the aspect of scientific research methodology, as well as sparking interest in aspects of transport economics of a methodological nature (De Maio & Vitetta, 2015).

An optimal transport system is understood as a system that fully and correctly secures the service of the existing transport needs (volume, types, area dispersion) with the lowest social labour input, while making rational use of the features and characteristics (technical, operational and economic) of individual transport modes. By the idea of this definition adopted in the literature, the issue of optimizing the transport system is reduced to the total and proper service of transport needs with the criterion of minimizing social labour input (Wong, et al., 2016).

**Optimization in transport** is a very broad concept that covers a variety of processes. Their aim is to improve the situation of the parties involved in transport (shippers, receivers, employees). Most often, aspects relating to optimization and forwarding can be found in the literature. Usually it is a question of reducing transport and delivery costs. In practice, however, the issue is multi-criteria. Less common criteria are also relevant, among which one can mention comfort, ecology, quality of transport services, customer satisfaction and even road wear and tear. Cost and time, however, are the dominant factors. Logistical processes should be considered from a strategic as well as an operational perspective. The strategic perspective refers to long-term planning and the design of a company's vision. The operational perspective deals with the current situation. These aspects of optimization should be linked to each other as well as supported by various logistics management systems. The topic of transport optimization, even if limited to cost minimization, involves a whole sequence of activities concerning the complete supply chain. The most important is the creation of an efficient distribution network, concerning the identification of the location of customers and their needs, which potentially change over time, and the optimal location of distribution centres, distribution terminals and warehouses. It is also important to choose the right fleet (size of vehicles, split between own and external fleets, as well as their capacities such as lifting, crane, refrigeration, cargo space), as an inadequate fleet can significantly reduce the potential of a transport company. The transport optimization process should also analyse different product flow scenarios, as well as a global identification of bottlenecks and profit-reducing factors for the company (www 10.1).



The advantages of a properly conducted optimization process include increased safety, better service quality and greater availability of goods and services. The integration of systems and the application of predefined standards, together with a skilful selection of optimization techniques, is helpful for national and regional transport and is also conducive to increasing the competitiveness of the enterprise. The most important benefits of optimization, when carried out correctly, are illustrated in Figure 10.1. The following figure shows the changes occurring through the implementation of optimization processes.

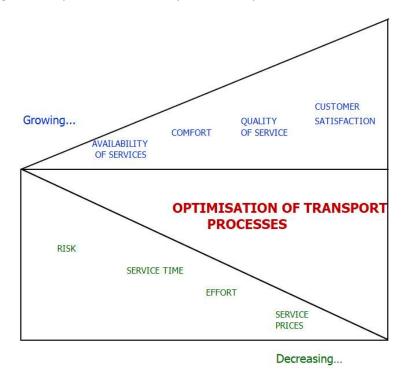


Figure 10.1. Changes resulting from the use of optimization of transport processes

Source: (Zajdel & Filipowicz, 2008)

Several optimization issues can be mentioned. The most important of them in the group of small shipments are:

- choice between indirect and direct carrier,
- distribution potential planning,
- consider local distribution.



The acceptance of transportation orders always raises the question of choosing between direct suppliers and the use of a distribution network. Making a decision regarding the transport of a shipment depends on terminal and delivery costs. This problem concerns the dimensions of the shipment. A large shipment and a long delivery distance encourage you to choose direct transport. If the shipment is a single shipment, the issue becomes easy to solve because it is enough to compare transportation costs using the terminal system. However, it should be borne in mind that combining a shipment with other items reduces the costs of indirect transport. Designing a distribution network for small parcels is a strategic issue. The total cost of the system can be calculated using the following formula (Milewski, 2011):

$$K_{CSD} = \sum_{j=1}^{n} K_{d-O_j} + K_{T_j} + K_{P_j}$$

where:

 $K_{CSD}$  – total cost of the distribution system,

 $K_{d-O_i}$ - transport and disposal costs of the  $\dot{F}$ th terminal,

 $K_{T_i}$  – terminal costs of the  $\dot{F}$ th terminal,

 $K_{P_i}$  – costs of linear transport of the  $\dot{F}$ th terminal,

n – number of terminals.

An operational problem is planning parcel transport routes within the framework of strategic arrangements. The total cost of transporting shipments along a given route can be expressed using the formula below (Milewski, 2011):

$$K_{CDL} = \sum_{k=1}^{O} K_{i,k} * d_k$$

where:

 $K_{CDL}$  – total cost of local distribution,

 $K_{i,k}$  – transportation cost (delivering or distributing) of shipments on route k,

 $d_k$  - route length k,

o – number of routes.



The main goal of optimization methods and models is to solve problems. The optimization criterion is usually the shortest possible transport time or the shortest route. This approach is sufficient assuming that the total cost depends directly on the length of the routes. Therefore, the route should be selected so that it is "as short as possible or the travel time along it is as short as possible." At this point, attention should be paid to the optimization model of the transport macrosystem (Milewski, 2011). Its task is to develop an appropriate number of indicators and measures necessary in the process of rational transport management during the logistic implementation of operational activities. The more precisely the model reflects the tested reality, the more effective the control capabilities are. Depending on the similarity, the optimization model can be used to directly create a business strategy in the transport services sector. The optimization model of the transport system provides methods and scientific tools for controlling the transport system. It can be written in the form of the following expression (Ficoń, 2010).

$$MDE_{ST} = \langle Z_{ST}, P_{ST}(t) \text{ II } G_{ST}, F_{ST}, H_{ST} \rangle \xrightarrow{maxSTO_{ST}} minS_{ST},$$

where:

 $Z_{ST}$  – a set of operational (logistic) resources of the ST system,

 $P_{ST}$  - a set of operational (logistics) processes of the ST system,

 $G_{ST}$  – a set of constraints and boundary conditions of the ST system,

 $F_{ST}$  – ST system operation criterion function,

 $H_{ST}$  – a set of acceptable operating schedules for the ST system,

 $S_{ST}$  – global costs of functioning of the ST transport macrosystem,

STO – logistic customer service standards by the transport sector.

The concept of optimization modeling of the transport system is presented in Figure 10.2.



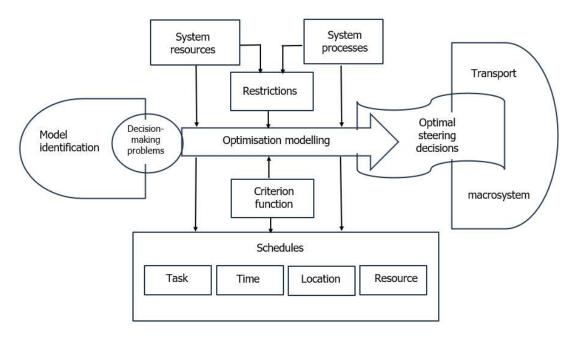


Figure 10.2. The concept of optimization modeling of the transport system

Source: (Ficoń, 2010)

There are, of course, other models to help solve optimization problems in transport There are many more transport optimization issues. There are differences between them depending on the company, its size and the products at its disposal.

### 10.3. Transport optimization issues in practice

One of the basic components of transport optimization is **route optimization**. This involves designing routes for the vehicles used to transport goods in order to optimize profits and customer service. For the company, this usually means a reduction in costs and transport time, while for customers it means lower costs and on-time delivery. These objectives are achieved by minimizing the distance travelled, being able to adapt transport schedules to changing conditions and new situations over time, synchronizing transport with warehouse operations, and being able to optimize frequently and quickly in order to update the current plan. Although the most common goal in transport tasks is to reduce transport costs to a minimum or strive to minimize distances, in the case of transporting products, mainly foodstuffs, that can quickly become out-of-date, products that quickly lose their useful



properties or those that are delivered according to the just-in-time principle due to limited storage capacities or excessive storage costs at the recipient's premises, or late deliveries by previous links in the supply chain, the priority objective is to minimise the lead time for all deliveries. Shorter delivery times make it possible to meet customers' expectations regarding delivery times, as well as to preserve the use value of the transported products, which can compensate for the expenses that result from the involvement of transport means. In the above cases, the most important consideration is therefore the reduction of the longest transport time in a given delivery system, i.e. the calculation of the shortest time in which deliveries could be completed (Gaspars-Wieloch, 2011).

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From an IT perspective, route optimization in transport and logistics is closely related to the **traveling salesman problem** (TSP) and the marshalling problem (VRP, Vehicle



Routing Problem). The traveling salesman problem is a classical combinatorial optimization problem, which involves planning the shortest and least expensive transportation route, passing through n specified sending and receiving points, with given travel costs between each pair of points. The VRP problem is a generalisation of the TSP problem. In it, it is possible to have multiple commuters (multiple vehicles), with the possibility of returning to the base multiple times before reaching all n locations. For both the TSP and the VRP problem, there are many generalisations and additional practical constraints, which can include time windows, sequential restrictions on the locations visited, different vehicle and driver capabilities, or capacity constraints, which are useful for delivery and collection (www\_10.2).

In order to solve the TSP task, it is necessary to specify: the level of product stock at each sending point, the volume of demand at each receiving point, as well as the transport costs from each sending point to each receiving point (Vinichenko, 2009). If only one product is involved, then the demand of the receiving points can be supplemented from one or more sending points. The intention of such a plan is to calculate the amount of products shipped from each sending point to each receiving point in order to minimise the total transport cost (Stachurski & Wierzbicki, 2001).

If the cost of the journey is directly proportional to the quantity transported, we are dealing with a linear transport task. Otherwise, if this condition is not met, the transport task becomes a non-linear task. One of the most popular optimization methods is linear programming (Silaen, et al., 2019; Gass, 2013). The greatest usefulness of this method is noted when creating a network of facilities, with the constraining conditions for the model being the size of demand and supply for production facilities, distribution centres or individual markets. With a given objective function, assuming, for example, a reduction in total cost, linear programming is helpful in creating an optimal facility deployment pattern that takes into account demand-supply constraints. Although the linear programming method is quite practical, there are limitations to its use, as the problem to be solved using it must be formulated deterministically, as well as the problem should be subjected to a linear approximation. In addition, the fixed and variable operating costs of logistics facilities cannot be taken into account in linear programming (Coyle, et al., 2002).



A large number of scientific papers can be found in the logistics literature on the theory and practice of organising an optimal transport system using various models and methods. Publications (Lai & Bierlaire, 2015; De Maio & Vitetta, 2015; Manley, Orr & Cheng, 2015; Vitetta, 2016) present route optimization studies according to the criterion of minimum delivery time.

In the articles (Hess, et al., 2015; Nyrkov, Sokolov & Belousov, 2015) methods based on alternative sampling were used to determine the optimal route. In contrast, the authors of publications (Zhilenkov, Nyrkov, & Cherniy, 2015; Omelianenko, et al., 2019; Tomashevskiy, 2007; Cheng & Wu, 2020; Zaychenko, 2014) have used fuzzy logic-based route modelling methods for transport systems. In the publications (Shang, et al., 2020; Shramenko & Shramenko, 2019) their authors, in order to plan the optimal route, used a heuristic model, while in the articles (Maleev, et al., 2019,; Skvortsov, Pshonkin & Luk'yanov, 2018,) a quantum model for determining the optimal route in transport systems was described.

The results of modelling the selection of optimal routes using **Global Positioning System** (GPS) data focused on trucks making long journeys can be consulted in publications (Khripach, et al., 2018; Navrodska, et al., 2019; Fialko, et al., 2020).

In the following, the individual steps in solving a transport task are presented in detail using a practical example of a transport problem with a time criterion for optimising the supply of a supermarket chain, which was described in the article (Gaspars-Wieloch, 2011).

# Problem characterisation of the transport problem with a time criterion for optimising the supply of a supermarket chain

In the transport optimization issue described, a supermarket chain distributed in different parts of the country is considered. A new range of goods is developed for each week, which, in addition to the always-on range, which includes groceries, drugstore products, are only sold to customers for six days from Monday to Saturday or until stocks run out. The week's offerings include white goods, paper products, clothing, toys, tools or gardening items, among others. Often, the product offer is tailored to the season and holidays such as Christmas, Easter, Valentine's Day, All Saints' Day, First Holy Communion, etc. The week's offerings are determined well in advance and the products they cover are stocked in wholesalers spread across



the country. Depending on the possibilities available to the suppliers, the different types of goods are delivered to the wholesalers a week in advance on different days (including Saturday mornings) before they go on sale. Wholesalers are required to prepare product kits for each shop. An example kit might include 20 napkins, 30 televisions, 40 buckets, 20 pairs of flip-flops, 30 pots, 20 teddy bears, 30 pairs of trousers, 60 hand creams, 50 balls and 40 notebooks.

As the kits may not be completed until the end of the week, and due to insufficient storage space in the supermarkets, the company is keen to deliver the week's range to all shops on the night of Sunday to Monday and spread them out on the shelves without delay.

Each lorry that leaves the warehouse delivers sets of products immediately to several or even a dozen supermarkets, forming a sector. The supermarkets in a sector are located fairly close to each other (e.g. in the same town). The time taken to supply a sector depends on which wholesaler a vehicle is diverted to it from. The key objective of the company is to minimise the longest delivery time.

#### Mathematical model of the transport task

The general form of a model describing a closed transport problem with a time criterion can be presented as follows (Gaspars-Wieloch, 2011):

$$\max_{x_y > 0} \{t_{ij}\} \to \min \tag{1}$$

$$\sum_{j=1}^{n} x_{ij} = a_i \quad (i-1, ..., m) \quad (2)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad (j = 1, ..., n)$$
 (3)

$$x_{i,i} \ge 0 \tag{4}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{5},$$

where:

 $x_{ij}$  — volume of goods transported from  $\emph{i-}$ th supplier to  $\emph{j-}$ th customer,





 $t_{ij}$  – transport time of the goods from  $\dot{F}$ th supplier to  $\dot{F}$ th consignee,

n – number of recipients,

m – number of suppliers,

 $a_i$  – supply of the *i*-th supplier,

 $b_i - j$ -th customer demand.

When there is no equality between total supply and total demand, the last formula (5) is not taken into account and the supply conditions (2) or demand conditions (3) turn into inequalities. The mathematical model described above is applicable to circumstances where the decision-maker does not assume other considerations, such as insufficient supply vehicles, a required minimum level of demand satisfaction or a difference in the nature of transport time and unloading time.

In scientific papers (Sikora, 2008), an algorithm was presented to solve the previously described transport problem. The subsequent steps of the course of action according to the said algorithm are listed below (Gaspars-Wieloch, 2011):

- 1. The first step is to determine the admissible base equation using the minimum matrix element method, known as the MEM method based on a table of times.
- 2. in the second step, it is important to determine the maximum delivery time (T<sup>k</sup>) for a particular solution based on the formula (6):

$$T^k = \max_{x_{ij} > 0} \{t_{ij}\},\tag{6}$$

where:

T<sup>k</sup> – maximum delivery time in the k-th iteration.

3. a cost table  $(c_{ij})$  for the k-th solution must be presented in turn according to formula (7):

$$c_{ij}^{k} = \begin{cases} 0 & t_{ij} < T^{k} \\ 1 & t_{ij} = T^{k} \\ & if \\ 10 & t_{ij} > T^{k} \end{cases}$$
 (7)



4. The next step is to check the optimality of the solution based on the cost table. In the case of non-negativity of the optimality criteria ( $\Delta_{ij}$ ) for all base routes, the procedure ends at this stage. If the optimality criteria are negative, follow the steps in step five.

$$\Delta_{ii}^k = c_{ii}^k - \alpha_i^k - \beta_i^k, \tag{8}$$

where:

 $\alpha_i^k$  ,  $\beta_j^k$  — dual variables, i.e. potentials in the  $\emph{k}\text{-th}$  iteration.

5. the acceptable base solution should be redetermined, taking into account the most negative optimality criterion, and then return to step two.

The above algorithm is based on the so-called potential method, which has been described in many publications, e.g. (Leonard, 1997). If there are other assumptions in a specific decision problem, concerning the limited packing capacity of the means of transport, then the procedure discussed should additionally refer to the principles that were adopted in the course of the procedure for a transport task with limited route capacity (Codeca & Cahill, 2022; Sanz & Escobar Gomez, 2013).

The algorithm can be used as a manual procedure when dealing with tasks with a small number of suppliers and customers. For problems of greater complexity, it is recommended to use an appropriate programme created for the algorithm in any programming language.

Another option for the procedure presented can be a developed optimization IT tool, an example of which is Solver, included in Microsoft Excel. However, it should be taken into account that the version of Solver has an impact on the type of tasks that can be solved. With each newer version, more possibilities are offered in terms of the number of conditions or variables in the task, the time required to solve the problem and the type of functions used. In the standard version of Solver, no "if", "max{}" or "min{}" function can be used. The "max" function appears in the mathematical model, described by formulas (1)-(5), so it seems that the task cannot be solved using the standard version of Solver. In order to carry out the calculations, an example with specific numerical data is considered below, for which an appropriate mathematical model has been formulated. The example concerns a supermarket chain comprising three wholesalers, P (in the south of the country), Z (in the west of the



country) and PW (in the north-east of the country), and 50 shops, divided into 8 differentiated sectors, denoted by letters (A, B, C, D, E, F, G and H). Based on the products transported, 18 kits can be drawn up by each wholesaler. The demand for kits per sector is as follows:

$$Z_A = 6$$
;  $Z_B = 7$ ;  $Z_C = 9$ ;  $Z_D = 6$ ;  $Z_E = 8$ ;  $Z_F = 5$ ;  $Z_G = 4$ ;  $Z_H = 5$ , whereby  $\sum_{j=1}^{8} 50$ .

The time taken by the delivery vehicles to serve each sector is formed by the travelling time from the wholesaler to the sector, which is not dependent on the number of shops in the sector, as well as the unloading time in the sector itself, which is dependent on the number of shops, as illustrated in Tables 10.1 and 10.2. It has been assumed that the transport time in a sector, which in practice is determined by the distance between the shops in the sector, is taken into account in the unloading times of the assortment in each shop. The aim is to minimise the delivery time taking the longest.

Table 10.1. Approximate journey time ( $t_{ii}^p$ , in hours )

Sectors Wholesalers	Α	В	С	D	E	F	G	Н
Р	9	6	3	3	6	9	12	7
Z	5	4	5	6	9	12	9	7
PW	8	5	11	5	3	3	4	3

Source: (Gaspars-Wieloch, 2011)

Table 10.2. Average unit unloading time ( $t_i^r$ , in hours)

Sectors	Α	В	С	D	Е	F	G	Н
$t_i^r$	1/3	1/2	1/3	2/5	1/2	2/5	1/4	2/5

Source: (Gaspars-Wieloch, 2011)

The example presented above with numerical data has a slightly higher degree of complexity than a standard transport problem with a time criterion. Therefore, the exact notation of the optimization task that applies to the example in question is only fragmentarily similar to the general mathematical model, as illustrated by formulas (9)-(21). The objective



function for minimising delivery time can be written as follows (Gaspars-Wieloch, H. in: Szymczak, M. (ed.), 2011, pp.17-18):

$$\max \left\{ (9 \min \left\{ x_{11}, 1 \right\} + \frac{1}{3} x_{11}), \left( 6 \min \left\{ x_{12}, 1 \right\} + \frac{1}{2} x_{12} \right), \left( 3 \min \left\{ x_{13}, 1 \right\} + \frac{1}{3} x_{13} \right), \left( 3 \min \left\{ x_{14}, 1 \right\} + \frac{1}{3} x_{13} \right), \left( 3 \min \left\{ x_{14}, 1 \right\} + \frac{1}{2} x_{14} \right), \left( 6 \min \left\{ x_{15}, 1 \right\} + \frac{1}{2} x_{15} \right), \left( 9 \min \left\{ x_{16}, 1 \right\} + \frac{2}{5} x_{16} \right), \left( 12 \min \left\{ x_{17}, 1 \right\} + \frac{1}{4} x_{17} \right), \left( 7 \min \left\{ x_{18}, 1 \right\} + \frac{2}{5} x_{18} \right), \left( 5 \min \left\{ x_{21}, 1 \right\} + \frac{1}{3} x_{21} \right), \left( 4 \min \left\{ x_{22}, 1 \right\} + \frac{1}{2} x_{22} \right), \left( 5 \min \left\{ x_{23}, 1 \right\} + \frac{1}{3} x_{23} \right), \left( 6 \min \left\{ x_{24}, 1 \right\} + \frac{2}{5} x_{24} \right), \left( 9 \min \left\{ x_{25}, 1 \right\} + \frac{1}{2} x_{25} \right), \left( 12 \min \left\{ x_{26}, 1 \right\} + \frac{2}{5} x_{26} \right), \left( 9 \min \left\{ x_{27}, 1 \right\} + \frac{1}{4} x_{27} \right), \left( 7 \min \left\{ x_{28}, 1 \right\} + \frac{2}{5} x_{28} \right), \left( 8 \min \left\{ x_{31}, 1 \right\} + \frac{1}{3} x_{31} \right), \left( 5 \min \left\{ x_{32}, 1 \right\} + \frac{1}{2} x_{32} \right), \left( 11 \min \left\{ x_{33}, 1 \right\} + \frac{1}{3} x_{33} \right), \left( 5 \min \left\{ x_{34}, 1 \right\} + \frac{2}{5} x_{34} \right), \left( 3 \min \left\{ x_{35}, 1 \right\} + \frac{1}{2} x_{35} \right), \left( 3 \min \left\{ x_{36}, 1 \right\} + \frac{2}{5} x_{36} \right), \left( 4 \min \left\{ x_{37}, 1 \right\} + \frac{1}{4} x_{37} \right), \left( 3 \min \left\{ x_{38}, 1 \right\} + \frac{2}{5} x_{38} \right) \right\} \rightarrow \min$$

$$(9)$$

The conditions that relate to sectoral demand are written as follows (Gaspars-Wieloch, 2011):

$$x_{11} + x_{21} + x_{31} = 6 ag{10}$$

$$x_{12} + x_{22} + x_{32} = 7 ag{11}$$

$$x_{13} + x_{23} + x_{33} = 9 ag{12}$$

$$x_{14} + x_{24} + x_{34} = 6 ag{13}$$

$$x_{15} + x_{25} + x_{35} = 8 ag{14}$$

$$x_{16} + x_{26} + x_{36} = 5 ag{15}$$

$$x_{17} + x_{27} + x_{37} = 4 ag{16}$$

$$x_{18} + x_{28} + x_{38} = 5 ag{17}$$

The conditions that apply to the supply of wholesalers are presented below (Gaspars-Wieloch, 2011):

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} \le 18 \tag{18}$$



$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} + x_{28} \le 18 \tag{19}$$

$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} + x_{37} + x_{38} \le 18 \tag{20}$$

The condition having to do with the integrability of the decision variables has the form (Gaspars-Wieloch, 2011):

$$x_{11}, x_{12}, \dots, +x_{38} \in N,$$
 (21)

where:

 $x_{11}$  – number of sets transported from warehouse P to supermarket A, :

 $x_{38}$  – number of sets transported from the PW warehouse to the supermarket H.

The example discussed is one of the rather convoluted ones, not only due to the lack of direct possibility to solve optimization problems containing the "max{}" and "min{}" functions in the standard version of Solver. Further complications include the two types of time introduced: the arrival time at each sector, and the discharge time at the sector. This was necessary because products transported by truck are not unloaded in one place, but in several shops. There is therefore a correlation between unloading times and the number of supermarkets served. The algorithm described earlier should not be directly applied to this type of problem.

# Design of the Excel spreadsheet in the presented example of a transport task

Figure 10.3 shows how to enter the data in the Microsoft Excel spreadsheet of the transport task example discussed. The cells with addresses C8-J10 are the fields where the optimal values of the decision variables ( $x_{11}$ ,  $x_{12}$ , ...,  $x_{38}$ ) will be shown. The summed field values of the corresponding columns of table C8-J10 were calculated in the row with the number 11. They represent the left-hand side of conditions (10)-(17). The formulated demand is included in the row with number 12. The added values from cell ranges C8:J8, C9:J9, C10:J10 are shown in the column labelled K. They contain the total number of sets transported from wholesalers P, Z, PW respectively, i.e. the left-hand side of supply constraints (18)-(20). The supply values of the individual wholesalers are shown in column L.

4 B	С	D	E	F	G	Н	1	J	K	L
Wholesalers and sectors	A	В	С	D	E	F	G	Н	Total at a given wholesaler	Supply
P				Č.		c.		Č.	0	18
Z									0	18
) PW	8						-		0	18
Sector total	0	0	0	0	0	0	0	0		
2 Demand	6	7	9	6	8	5	4	5		
3										
5	9	6	3	3	6	9	12	7	1	
Travel time to sector	5	4	5	6	9	12	9	7		
3	8	5	11	5	3	3	4	3		
			100%							
Unloading time	0.33	0.50	0,33	0.40	0.50	0.40	0.25	0.40		
	0	0	0	0	0	0	0	0		
	0	0	0	0	0	0	0	0	:.	
Total unloading time	0	0	0	0	0	0	0	0	1	
7	1 1									
Total journey time	9	6	3	3	6	9	12	7	1	
(fixed) and unloading	5	4	5	6	9	12	9	7		
time (variable)	8	5	11	5	3	3	4	3		
3										
	0	0	0	0	0	0	0	0		
Base and non-base	0	0	0	0	0	0	0	0		
routes	0	0	0	0	0	0	0	0		
)						-				
	0	0	0	0	0	0	0	0		Objective function
lotal time on base	0	0	0	0	0	0	0	0		0.0
routes	0	0	0	0	0	0	0	0		100.000

Figure 10.3. Data entered into the spreadsheet in this example of a transport task

Source: (Gaspars-Wieloch, 2011)

Rows 16-45 present a summary of the parameters and formulas needed to determine the objective function. The data from Tables 10.1 and 10.2 are presented in rows 16-18 and 20. The total unloading time from the i-th wholesaler to the j-th sector in rows 24-26 has been calculated by multiplying the unit unloading time from row 20 by the number of delivered sets in rows 8-10, as shown in Figure 10.4.

24		=C20*C8	=D20*D8	=E20*E8	=F20*F8	=G20*G8	=H20*H8	= 20* 8	=J20*J8
25	Total unloading time	=C20*C9	=D20*D9	=E20*E9	=F20*F9	=G20*G9	=H20*H9	= 20* 9	=J20*J9
26		=C20*C10	=D20*D10	=E20*E10	=F20*F10	=G20*G10	=H20*H10	= 20* 10	=J20*J10

Figure 10.4. Calculation of total unloading time

Source: compiled on the basis of (Gaspars-Wieloch, 2011)

The method of determining the total travel time and unloading time in rows numbered 30-32 is shown in Figure 10.5.

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30	Total inurnay time (fixed) and unleading time	=C16+C24	=D16+D24	=E16+E24	=F16+F24	=G16+G24	=H16+H24	= 16+ 24	=J16+J24
31	lotal journey time (fixed) and unloading time (variable)	=C17+C25	=D17+D25	=E17+E25	=F17+F25	=G17+G25	=H17+H25	=117+125	=J17+J25
32	(valiable)	=C18+C26	=D18+D26	=E18+E26	=F18+F26	=G18+G26	=H18+H26	=118+126	=J18+J26

Figure 10.5. Calculation of total unloading time

Source: compiled on the basis of (Gaspars-Wieloch, 2011)

The purpose of writing  $min\{x_{ij},1\}$  in formula (9) is to extract the underlying routes. In the standard version of Solver, it is not possible to solve tasks where the function 'min{}' is present. Therefore, the base routes must be determined differently. If the quotient in formula (22) has a value close to the number 1, then this route can be referred to as the base route. If, on the other hand, the quotient is equal to zero, then the transport on the examined route will not take place, as shown in Figure 10.6.

$$\frac{x_{ij}}{x_{ij} + 0,00001} \tag{22}$$

1				E .	I.			I .	
36	Complete the Albertain	=C8/(C8+0,00001)	=D8/(D8+0,00001)	=E8/(E8+0,00001)	=F8/(F8+0,00001)	=G8/(G8+0,00001)	=H8/(H8+0,00001)	=18/(18+0,00001)	=J8/(J8+0,00001)
37	Base and non-base routes	=C9/(C9+0,00001)	=D9/(D9+0,00001)	=E9/(E9+0,00001)	=F9/(F9+0,00001)	=G9/(G9+0,00001)	=H9/(H9+0,00001)	=19/(19+0,00001)	=J9/(J9+0,00001)
38		=C10/(C10+0,00001)	=D10/(D10+0,00001)	=E10/(E10+0,00001)	=F10/(F10+0,00001)	=G10/(G10+0,00001)	=H10/(H10+0,00001)	=110/(110+0,00001)	=J10/(J10+0,00001)

Figure 10.6. Determination of base and non-base routes

Source: compiled on the basis of (Gaspars-Wieloch, 2011)

Using the method described, only the times for the base routes can be included in the final step. The formulae for calculating the total time on the base routes can be found in Figure 10.7. and can be found in rows 43-45. The cells in lines 43-45 are the successive arguments of the {max} function that appears in formula (9). The objective function itself is contained in cell L44.

			- y	U	4	1		- L	1
43	200 take 100 kilone 11	=C36*C30	=D36*D30	=E36*E30	=F36*F30	=G36*G30	=H36*H30	=136*130	=J36*J30
44	Total time on base routes	=C37*C31	=D37*D31	=E37*E31	=F37*F31	=G37*G31	=H37*H31	= 37* 31	=J37*J31
45		=C38*C32	=D38*D32	=E38*E32	=F38*F32	=G38*G32	=H38*H32	=138*132	=J38*J32

Figure 10.7. Determination of total cost on base routes

Source: compiled on the basis of (Gaspars-Wieloch, 2011)

#### Solving the problem in the presented transport task



To obtain the optimal solution, the Solver window still needs to be completed (Figure 10.8). The non-negativity of variables must be selected in the options, and then the "Solve", "Store Solution" and "Ok" commands selected.

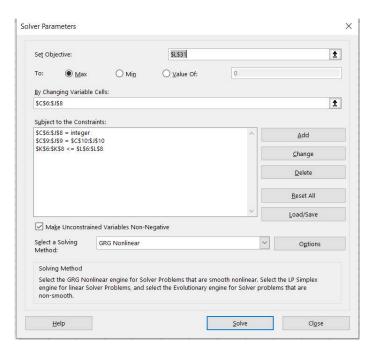


Figure 10.8. Formulas in the Solver window

Source: compiled on the basis of (Gaspars-Wieloch, 2011)

The resulting calculations are presented in Figure 10.9. However, they should be approached with caution, as the creation of the worksheet only managed to avoid the use of functions of the type "min{}", while the function "max{}"still remained. The results should therefore be looked at more closely. The transport of products on the route that connects wholesaler P to sector F currently has the longest duration, i.e. 9.8 hours, and needs to serve two shops ( $x_{16}=2$ ). Therefore, an attempt should be made to find a more favourable scheduling of deliveries by adding the condition  $x_{16} \le 1$ , i.e.  $$H$6 \le 1$ . This will reduce the transit and discharge times to a maximum of 9.8-0.4=9.4 hours, resulting in a more favourable solution.

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4	В	С	D	E	F	G	Н	1.	J	K	L
1			7107-4					,			
5	Wholesalers and sectors	А	В	С	D	E	F	G	Н	Total at a given wholesaler	Supply
6	Р	0,0	3,0	7,0	2,0	3,0	2,0	0,0	0,0	17	18
7	Z	6.0	1.0	2.0	4.0	0.0	0.0	1.0	1.0	15	18
8	PW	0.0	3.0	0.0	0.0	5.0	3.0	3.0	4.0	18	18
9	Sector total	6	7	9	6	8	5	4	5		2 0.000
10	Demand	6	7	9	6	8	5	4	5		
11											
12		9	6	3	3	6	9	12	7		
13	Travel time to sector	5	4	5	6	9	12	9	7		
14		8	5	11	5	3	3	4	3		
15				Ï					í	i i	
16	Unloading time	0,33	0,50	0,33	0,40	0,50	0,40	0,25	0,40		
17											
18	a mount - aun mouse	0	1,5	2,31	0,8	1,5	0,8	0	0		
19	Total unloading time	1,98	0,5	0,66	1,6	0	0	0,25	0,4		
20		0	1,5	0	0	2,5	1,2	0,75	1,6	8	
21											
22	Total journey time	9	7,5	5,31	3,8	7,5	9,8	12	7		
23	(fixed) and unloading	6,98	4,5	5,66	7,6	9	12	9,25	7,4		
24	time (variable)	8	6,5	11	5	5,5	4,2	4,75	4,6		
25			-								
26 27	Base and non-base	0	1	1	1	1	1	0	0		
27	routes	1	1	1	1	0	0	1	1	8	
28	routes	0	1	0	0	1	1	1	1	2	
29											
30	Total time on base	0,0	7,5	5,3	3,8	7,5	9,8	0,0	0,0		Objective function
31	routes	7,0	4,5	5,7	7,6	0,0	0,0	9,2	7,4		9,8
32	Toutes	0,0	6,5	0,0	0,0	5,5	4,2	4.7	4,6		

Figure 10.9. First solution to an optimization task

Source: (Gaspars-Wieloch, 2011)

The second plan is included in Figure 10.10. In it, it is worth including the condition  $x_{28} \le 2$ , i.e.  $\$/\$9 \le 2$ , as it will then contribute to reducing the delivery time on the Z-H route by at least 0.4 hours (the unloading time for products in the H sector will then be 8.2-0.4=7.8 hours).

4	В	С	D	E	F	G	Н	1	J	K	L
5	Wholesalers and sectors	A	В	С	D	E	F	G	Н	Total at a given wholesaler	Supply
6	Р	0,0	3,0	8,0	1.0	3,0	0.0	0,0	1,0	16	18
7	Z	6.0	2.0	1,0	4.0	0.0	0.0	0,0	3,0	16	18
8	PW	0,0	2,0	0,0	1,0	5,0	5,0	4.0	1,0	18	18
9	Sector total	6	7	9	6	8	5	4	5	7.7	
10	Demand	6	7	9	6	8	5	4	5	27	
11 29											
30	Total discount base	0,0	7,5	5,6	3,4	7,5	0,0	0,0	7,4		Objective function
31 32	Total time on base	7,0	5,0	5,3	7,6	0,0	0,0	0,0	8,2		8,2
32	routes	0,0	6,0	0,0	5,4	5,5	5,0	5,0	3,4		29(02)

Figure 10.10. Second solution to an optimization task

Source: (Gaspars-Wieloch, 2011)



The third solution is illustrated in Figure 10.11. The time on the Z-H route actually decreased to 7.5 hours, but the longest time was recorded on the P-E route (8 hours). One might be tempted to see if adding the criterion  $x_{15} \le 3$ ,  $czyli \$G\$8 \le 3$  would improve the final result?

4	В	С	D	E	F	G	Н	Ť	J	К	L
5	Wholesalers and sectors	Α	В	С	D	E	F	G	н	Total at a given wholesaler	Supply
6	Р	0,0	2,0	8,0	4,0	4.0	0.0	0,0	0,0	18	18
7	Z	6,0	4,0	1,0	2,0	0,0	0,0	0,0	2,0	15	18
8	PW	0,0	1,0	0,0	0,0	4,0	5,0	4.0	3,0	17	18
9	Sector total	6	7	9	6	8	5	4	5		
10	Demand	6	7	9	6	8	5	4	5		
11 29		_									
30	Total Control Land	0,0	7,0	5,6	4,6	8,0	0,0	0,0	0,0		Objective function
30 31 32	Total time on base	7,0	6,0	5,3	6,8	0,0	0,0	0,0	7,8	0	8,0
32	routes	0,0	5,5	0,0	0,0	5,0	5,0	5,0	4,2	0	111

Figure 10.11. The third solution to the optimization task

Source: (Gaspars-Wieloch, 2011)

In the fourth solution, shown in Figure 10.12, the longest delivery time is already only 7.5 hours. After introducing constraints on the routes, which now determine the value of the objective function:  $x_{12} \le 2$ ,  $czyli \ D\$8 \le 2 \ i \ x_{15} \le 2$ ,  $czyli \ \$G\$8 \le 2$ .

Figure 10.13 shows the fifth most optimal solution. Even if further constraints are added, they will no longer improve delivery times.

A	В	С	D	E	F	G	Н	1	J	K	L
5	Wholesalers and sectors	А	В	С	D	E	F	G	Н	Total at a given wholesaler	Supply
3	Р	0,0	3,0	4,0	6,0	3,0	0.0	0,0	1,0	17	18
7	Z	6.0	3,0	5,0	0,0	0.0	0.0	0.0	1.0	15	18
3	PW	0,0	1,0	0,0	0,0	5,0	5,0	4.0	3,0	18	18
)	Sector total	6	7	9	6	8	5	4	5		
0	Demand	6	7	9	6	8	5	4	5		
9											
0 1 2	The state of the s	0,0	7.5	4.3	5.4	7,5	0.0	0.0	7.4		Objective function
1	Total time on base	7,0	5,5	6,6	0,0	0,0	0.0	0,0	7.4		7,5
2	routes	0,0	5,5	0,0	0,0	5,5	5.0	5,0	4.2	.00	<b>5</b> 1501

Figure 10.12. Fourth solution to an optimization task

Source: (Gaspars-Wieloch, 2011)



4	В	С	D	E	F	G	Н	1	J	К	L
5	Wholesalers and sectors	A	В	С	D	E	F	G	Н	Total at a given wholesaler	Supply
6	Р	0,0	2,0	4,0	6,0	2,0	0,0	0,0	1,0	15	18
7	Z	6,0	5,0	5,0	0,0	0.0	0,0	0,0	1,0	17	18
8	PW	0,0	0,0	0,0	0,0	6.0	5.0	4.0	3.0	18	18
9	Sector total	6	7	9	6	8	5	4	5		
10	Demand	6	7	9	6	8	5	4	5	71	
11											
30	T-1-10	0,0	7,0	4,3	5,4	7,0	0,0	0,0	7,4		Objective function
30 31 32	Total time on base	7,0	6,5	6,6	0,0	0,0	0,0	0,0	7,4		7,4
32	routes	0,0	0,0	0,0	0,0	6.0	5,0	5.0	4,2		200

Figure 10.13. Fifth solution to an optimization task

Source: (Gaspars-Wieloch, 2011)

The assumption for the optimal solution to the task is that the longest delivery time of 7.4 hours will be recorded on the two routes: P-H and Z-H. Each sector will be supplied with kits in line with reported demand. The supply of PW wholesalers will be used to the maximum. 13 trucks will be required to supply the supermarket chain.

Comparing the results obtained in the fourth and fifth solutions, one could actually end up implementing the fourth option due to the slight difference in time (7.5-7.4=0.1 h). It is worth noting here that in the fourth plan, in addition to a slightly longer delivery time, as many as 14 goods vehicles would have to be dispatched. The result obtained is of course not the only optimal solution. Different simulations may lead to different conclusions.

The analysis of the scientific works that were mentioned in the earlier sections of this article shows that the researchers' studies use different analytical approaches to the organisation of freight traffic and the modes of operation of facilities, as well as the modes of operation of individual elements and parts of logistics systems. This makes it possible to choose a method with which to optimize the transport system, which is an extremely important component of logistics processes in a company, affecting the profitability of the company.

# **Chapter Questions**

1. What are the main transport policy problems related to the optimization of the transport system?

- 2. What are the main goals of route optimization for the company and recipients?
- 3. What is the traveling salesman problem (TSP) and how is it related to route optimization?

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